Lesson 4: Signal transmission & Noise

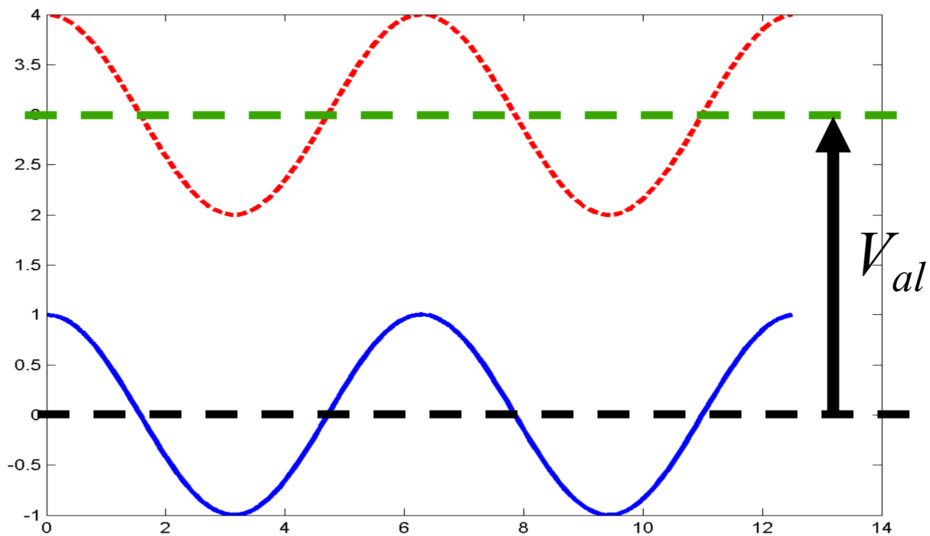
- **Signal Transmission**
  - Coupling scheme
  - Transmission line
  - Termination & impedance matching
- **Noise**
  - White noise
  - Pink noise
  - Lock-in amplifier: measuring modulated signal buried into noise

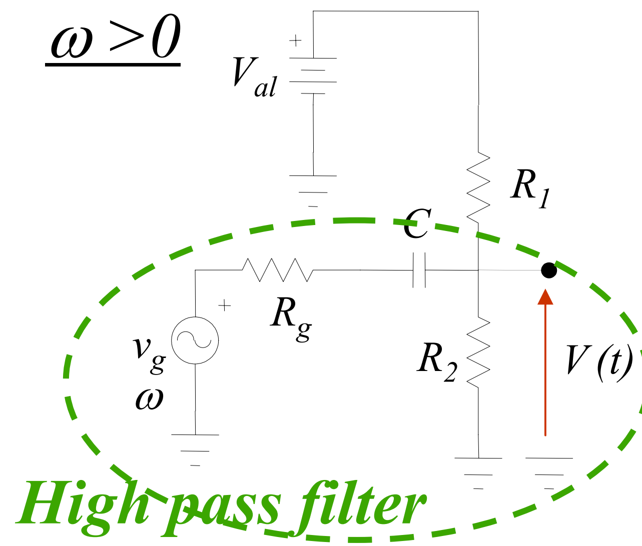
Signal transmission

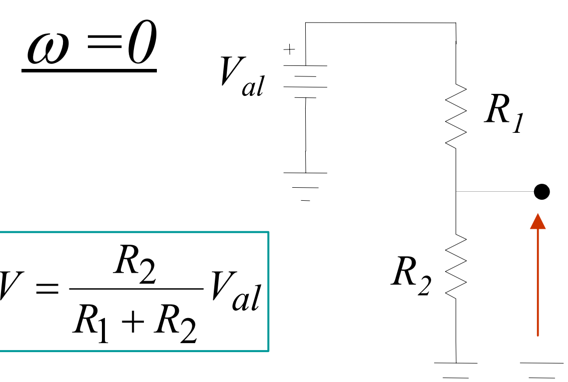
- There is often a need to propagate a signal over long distance
- This is accomplished with a transmission line in electronics
- Fancier system convert electronic signal into an optical pulses and use fiber to propagate signal over very long distance (e.g. communication cable under Atlantic ocean)
- The fundamental questions are:
  - How do we “inject” a signal into a transmission line
  - How do we model the effect of long transmission line on the electrical signal
  - How do we “terminate” the transmission line to get an handle on the signal and avoid reflections and/or interferences

Capacitive Coupling

- A simple RC filter is used to block DC component
- An DC voltage source offset the average of V(t).



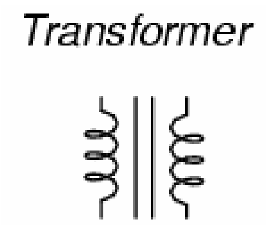


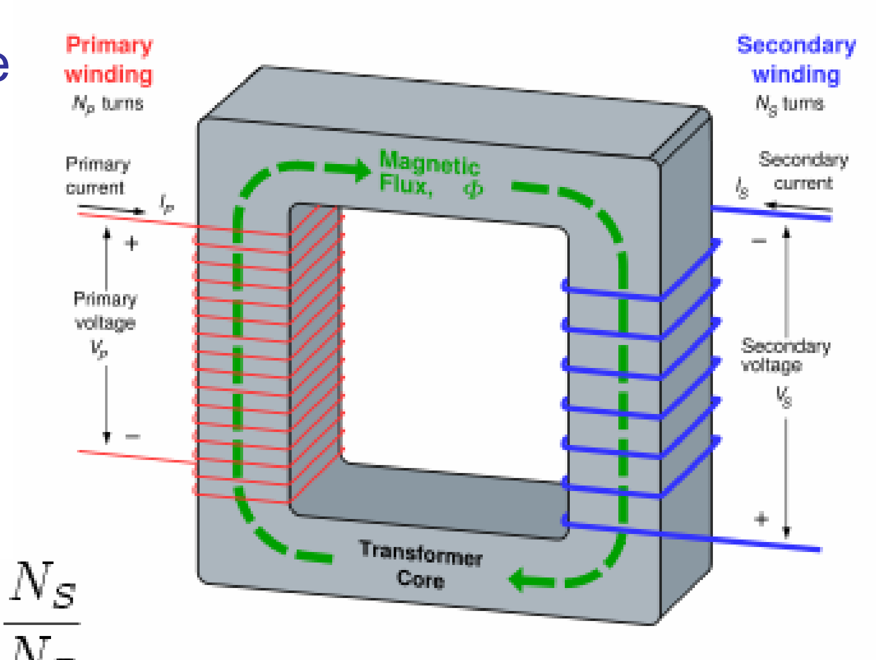


$$\rightarrow V = \frac{R_2}{R_1 + R_2} V_{al}$$

Inductive Coupling: transformer

- Transformer can be used as “couple





- Transformer equation

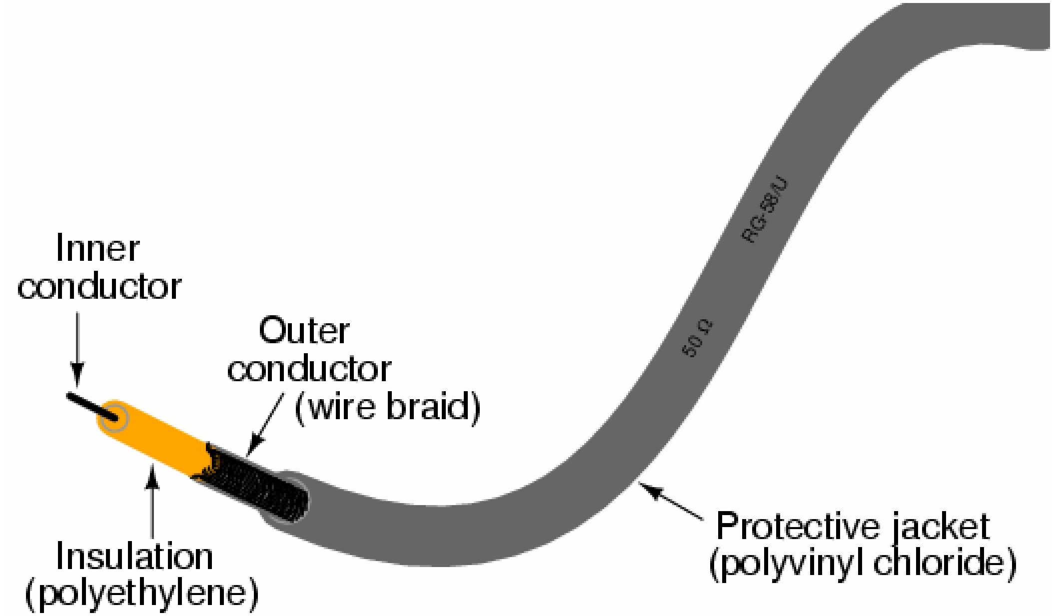
$$\left.\begin{aligned} v_P &= N_P \frac{d\Phi}{dt} \\ v_S &= N_S \frac{d\Phi}{dt} \end{aligned}\right\} \quad \frac{v_P}{v_S} = \frac{N_P}{N_S} \qquad \frac{i_P}{i_S} = \frac{N_S}{N_P}$$

- Output impedance of the system:

$$Z_{output} = n^2 Z_{input}, \quad n \equiv \frac{N_P}{N_S}$$

Transmission line: coaxial cable

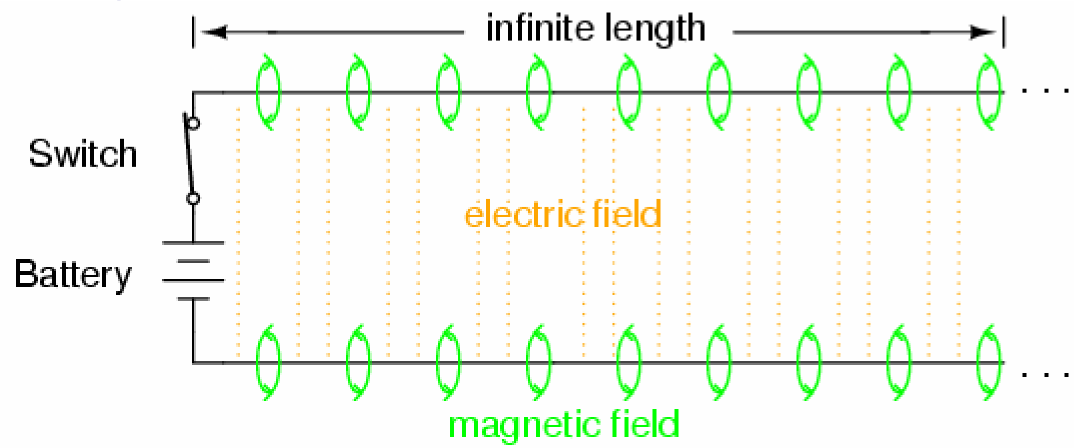
- You already used this cable in the Lab (to connect oscilloscope or frequency generator to you circuits)

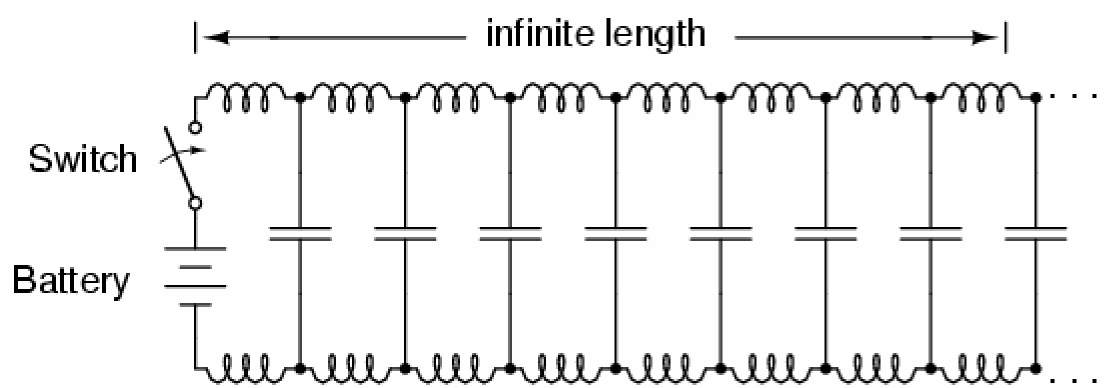


- How do we model this cable?

Transmission line: coaxial cable

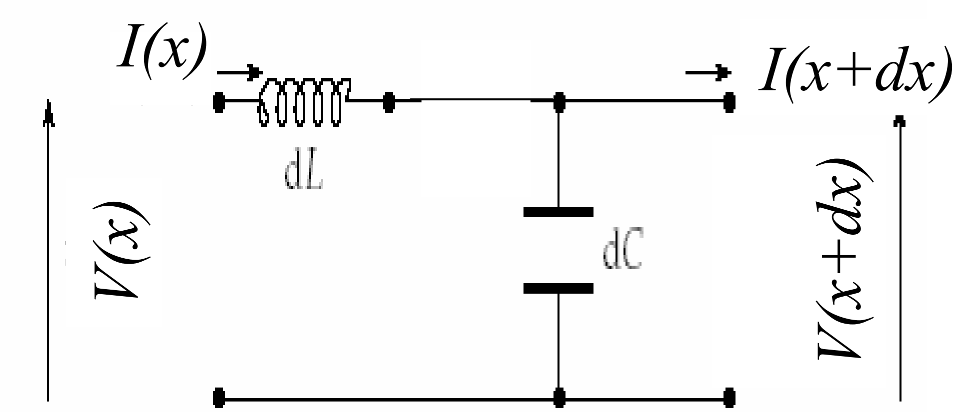
- Coaxial cable can be model as a chain of inductance and capacitance (there is also some resistance of course)





Transmission line: "Wave equation"

- Introduce the capacitance ***dC=c₀dx*** and inductance ***dL=l₀dx*** per unit of length.
- We can apply Kirchoff's voltage and current low to on LC cell of the circuit



$$\begin{cases} I(x) = I(x+dx) + c_0 \dfrac{dV(x)}{dt} \\ V(x) = V(x+dx) + l_0 \dfrac{dI(x+dx)}{dt} \end{cases}$$

$$\left.\begin{aligned} \frac{dI}{dx} &= -c_0 \frac{dV}{dt} \\ \frac{dV}{dx} &= -l_0 \frac{dI}{dt} \end{aligned}\right\} \quad \boxed{\frac{d^2V}{dx^2} - l_0 c_0 \frac{d^2V}{dt^2} = 0}$$

wave equation

Solution of wave equation

- So the wave equation is satisfied by both *I* and *V*

$$\frac{d^2}{dx^2}\begin{pmatrix} V \\ I \end{pmatrix} - l_0 c_0 \frac{d^2}{dt^2}\begin{pmatrix} V \\ I \end{pmatrix} = 0$$

- This are wave equations (in electromagnetism both scalar and vector potentials associated to an e.m. wave obey this equation)

- The $l_0 c_0$ quantity has the dimension [$L^{-2}.T^{2}$]: $v \equiv \frac{1}{\sqrt{l_0 c_0}}$

- The solution of the wave equation are of the form:

$$I(x,t) = I_0 e^{i(\omega t - kx)} + I_1 e^{i(\omega t + kx)}$$

$$V(x,t) = Z[I_0 e^{i(\omega t - kx)} - I_1 e^{i(\omega t + kx)}]$$

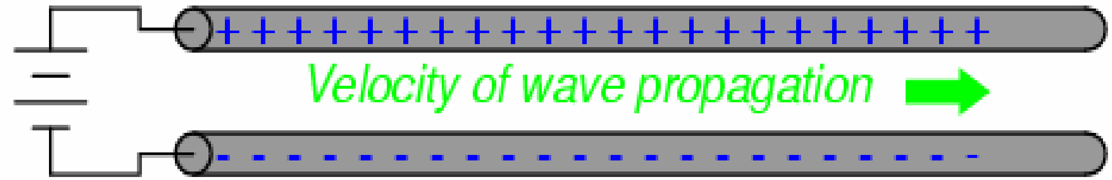
Forward TW

Backward TW

Coaxial cable impedance

- Plugging traveling wave solution into 1st differential equation directly gives a relation between *V* and *I*

$$Z \equiv \sqrt{\frac{l_0}{c_0}}$$



- A cable can be treated as an element with the above impedance.

$$\text{Velocity factor} = \frac{v}{c} = \frac{1}{\sqrt{\varepsilon_r}}$$

Where,

v = Velocity of wave propagation

c = Velocity of light in a vacuum

ε_r = Relative permittivity of insulation between conductors

- **Impedance of the cable is independent of its length!**

- **signal propagates with velocity close to *c* (actually *c/n*)**

Termination: Impedance matching

- Suppose at *x=L* we connect a resistor with resistance R then, at x=L we have

$$V(x=L,t)=RI(x=L,t)$$
$$\Rightarrow Z[I_0e^{i(\omega t-kL)}-I_1e^{i(\omega t+kL)}]=R[I_0e^{i(\omega t-kL)}+I_1e^{i(\omega t+kL)}]$$
$$\Rightarrow I_1=\frac{R-Z}{R+Z}I_0e^{-2ikL}$$

- So the current and voltage take the form

$$I(x,t)=I_0[e^{i(\omega t-kx)}-re^{i(\omega t+kx)}]$$
$$V(x,t)=ZI_0[e^{i(\omega t-kx)}+re^{i(\omega t+kx)}]$$

with *r* being a reflection coefficient $r\equiv\frac{I_1}{I_0}$

- *r=0* if Z=R (impedance matching)
- **Situation more complicated in practice since signal not really monochromatic**

Termination: Impedance matching

- If **termination is open** *($R \to \infty$)* then *$r = \exp(-i2kL)$* and the voltage becomes

$$V(x,t) = 2V_0 \cos(\omega t - kL) \cos k(x - L)$$

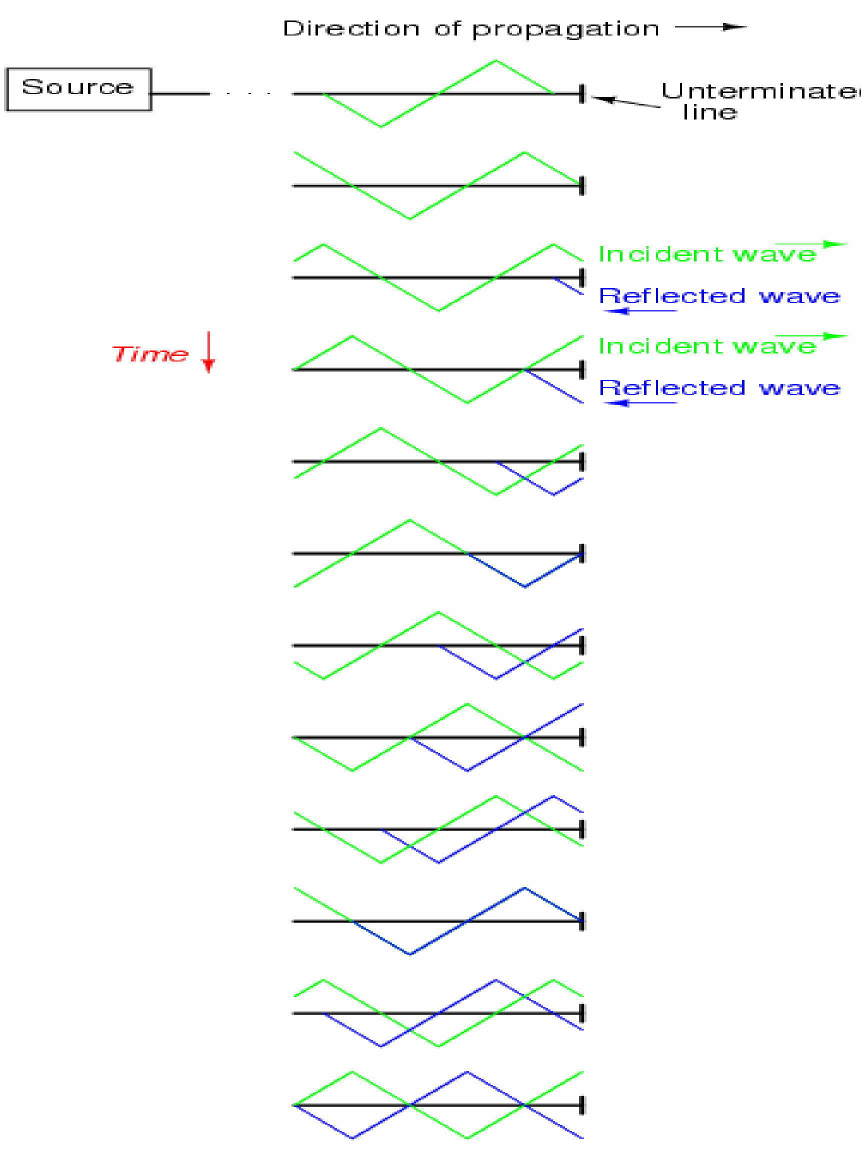
this is a stationary wave!

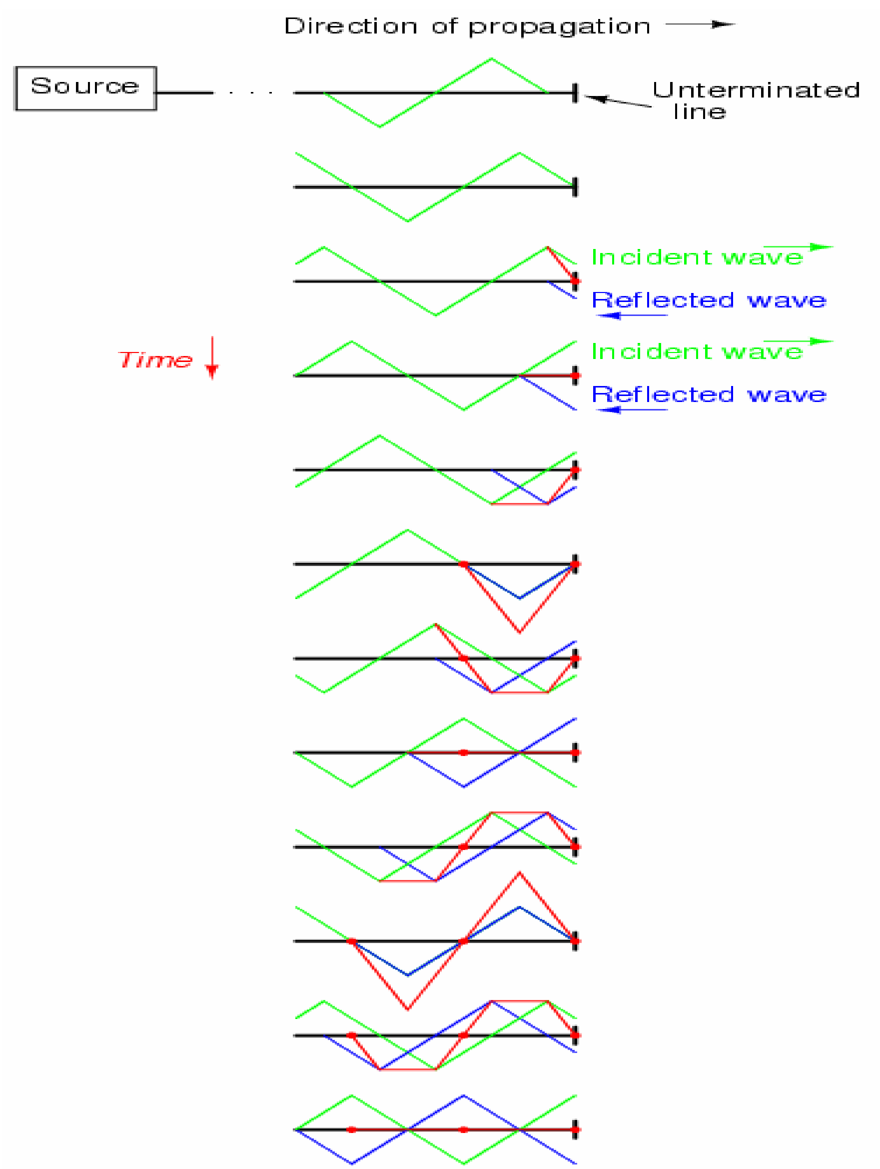
- If **termination is closed** *(short circuit: $R=0$)* then *$r = -\exp(-i2kL)$* and the voltage becomes

$$V(x,t) = -2V_0 \sin(\omega t - kL) \sin k(x - L)$$

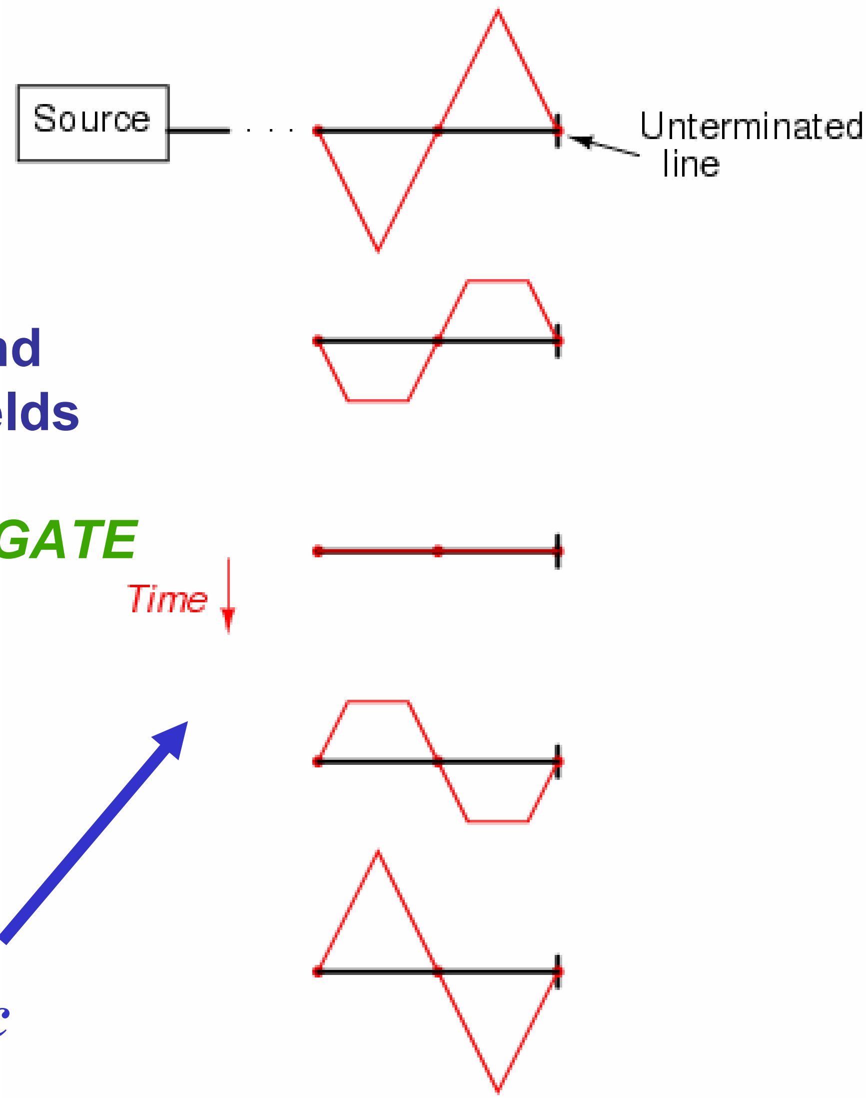
this is again a stationary wave!

Termination: case of open end





Termination: case of open end (CNT'D)



- **Combination of a forward and backward traveling wave yields a standing wave:** ***SIGNAL DOES NOT PROPAGATE***

This is a standing wave as one would find in a resonant electromagnetic cavity

Noise: Thermal Noise (or Johnson–Nyquist noise)

- In a conductor electron have random motion due to temperature.
- The **power spectral density** (=mean square voltage per unit of frequency in **V²/Hz**) associated to this thermal noise is:

$$\frac{d\langle V^2\rangle}{df} = 4kTR\frac{hf}{kT}\frac{1}{e^{\frac{hf}{kT}}-1} \approx 4kTR$$

"Low" frequency (f<1 GHz) approximation

R: resistance,
k: Boltzmann const.
h: Planck const.
T: temperature [K]
f: frequency [Hz]

- So the rms voltage noise is $\langle V^2\rangle^{1/2} = 2\sqrt{kTR\Delta f}$

Frequency bandwidth

- and the rms current noise is $\langle I^2\rangle^{1/2} = 2\sqrt{\frac{kT\Delta f}{R}}$

Noise: Shot Noise

- Shot noise comes from the fact that current is generated by a FINITE amount of discrete charge carried by electrons

- The current fluctuate with a probability following Poisson's distribution with variance

$$\left\langle I^2 \right\rangle^{1/2} = \sqrt{2\,|\,e\,|\left\langle I \right\rangle \Delta f}$$

e: electron charge
Δf: frequency bandwidth [Hz]

- Through a resistor this introduces a power fluctuation

$$\left\langle P^2 \right\rangle^{1/2} = 2R\,|\,e\,|\left\langle I \right\rangle \Delta f$$

Noise: Flicker Noise (1/f)

- Shot and Thermal noise are white noise (no frequency dependence on power)

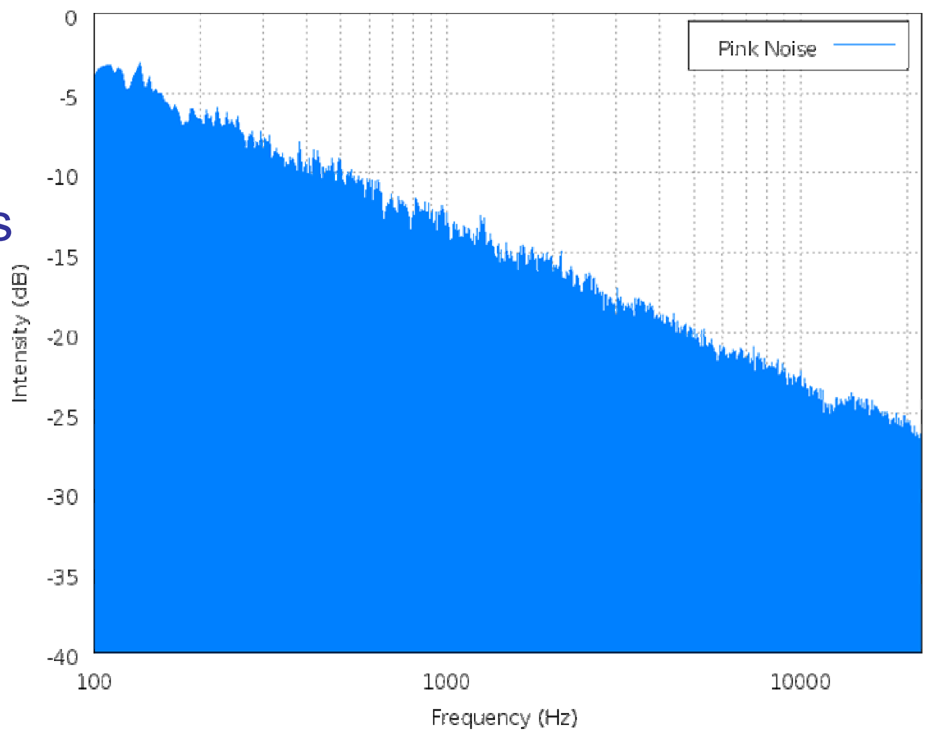
- Colored noise also exit typically

$$\frac{d\left\langle V^2\right\rangle}{df} \propto \frac{1}{f^{\alpha}}$$

- When α=1 noise is refered to as "**pink**" noise or ***1/f*** noise

- In electronics pink noise is due to a variety of cause: impurity, carrier/hole recombination, …

- Flicker noise appear for instance in resistors and transistors



How do we deal with and characterize noise?

- Work with small bandwidth system, optimize the bandwidth for the signal
- Can use RC filter to cut both hand of the spectrum
- A figure-of-merit to quantify noise compared to the main signal is the S/N ratio $$\left(S/N\right)_{Db} \equiv 10\log_{10}\left(\frac{S}{N}\right)$$
- Most of the time engineers like to express this S/N ratio as the unit of S/N is this latter expression is **Decibel** (Symbol Db)
- Some practical "conversion": $S/N \equiv \dfrac{P_{signal}}{P_{noise}}$

$$S/N = 10 \Leftrightarrow \left(S/N\right)_{Db} = 10\ Db$$

$$S/N = 2 \Leftrightarrow \left(S/N\right)_{Db} \approx 3\ Db$$

$$S/N = 1/2 = 0.5 \Leftrightarrow \left(S/N\right)_{Db} \approx -3\ Db$$

How do we deal with and characterize noise?

- The effect of a system on the noise is quantified by a noise factor:

$$F = 10\log_{10}\frac{(S/N)_{OUT}}{(S/N)_{IN}}$$

- Design system such that F<1 (to reduce noise)

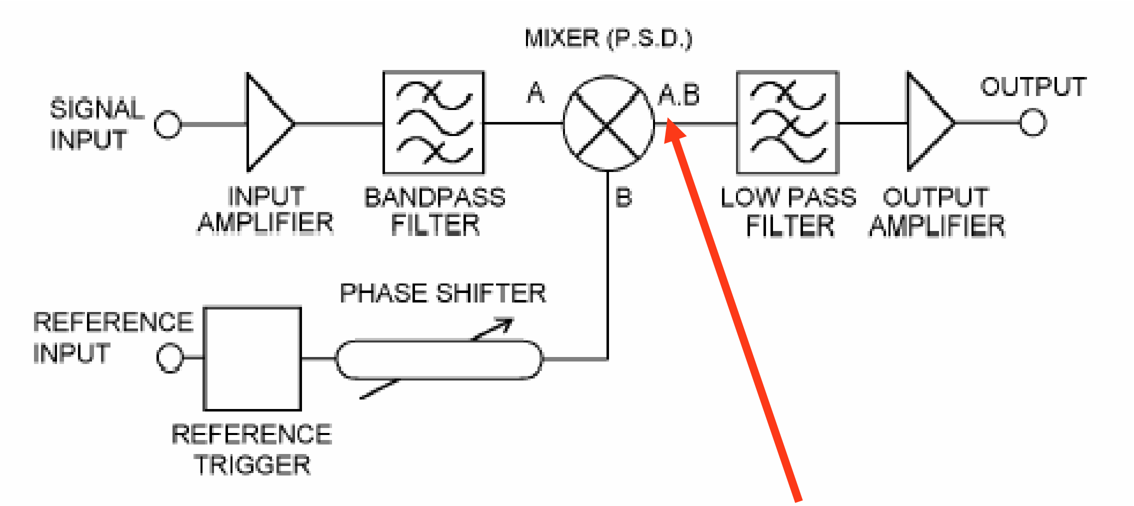
- Sometime one has to “live” with noise and use some experimental technique to recover the signal buried in the noise; very typical in low light detection experiments for instance…

  This is accomplished by using a so-called Lock-in amplifier

Detection of ultra-low signal buried in noise: The lock-in amplifier

$$A = A\cos(\omega t) + N$$

$$B = B\cos(\omega t + \varphi)$$



$$AB = \hat{A}\hat{B}\cos(\omega t)\cos(\omega t + \varphi) + \hat{A}N\cos(\omega t + \varphi)$$

$$= \frac{\hat{A}\hat{B}}{2}[\cos(2\omega t + \varphi) + \cos(\varphi)] + \hat{A}N\cos(\omega t + \varphi)$$

What remains after low pass filter

Lock-in amplifier: a numerical example

A Frequency-Domain Description of a Lock-in Amplifier

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The basic principles behind the operation of a lock-in amplifier are described. Particular emphasis is placed on looking at the frequency components of the signal present at the various stages of the lock-in during a typical measurement. The description presented here has been used successfully at Oberlin College to explain lock-in operation to students.

American Journal of Physics **62** (2) 129-133 (Feb. 1994).

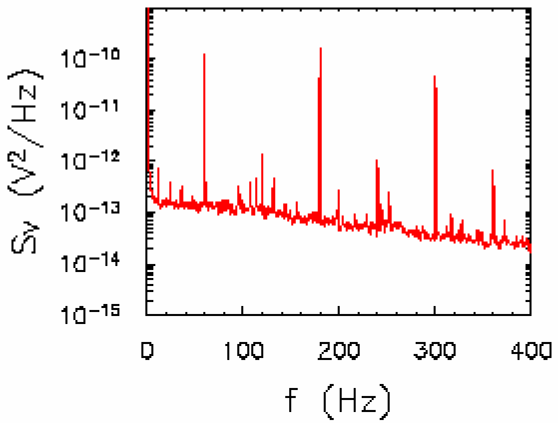

Figure 3. Graph of the power spectral density of the input signal. The desired signal is the "spike" at 200 Hz. Other spikes are due to unwanted pickup from the 110 VAC line at 60 Hz and its harmonics.

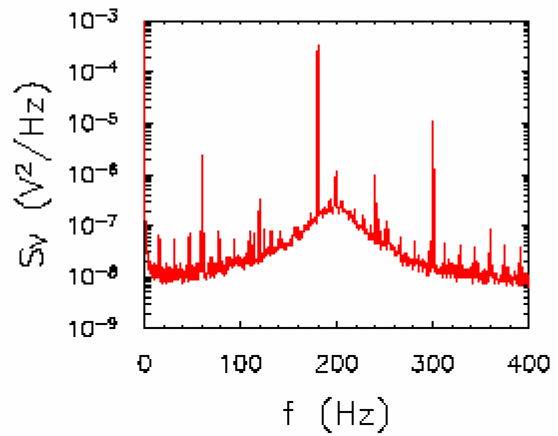

Figure 4. Power spectral density of the output of the AC amplifier stage. The bandpass character allows the desired frequency of 200 Hz to be amplified while frequencies well away from 200Hz are attenuated. In particular we have reduced the effects of line frequency pickup and the 1/f noise near DC.

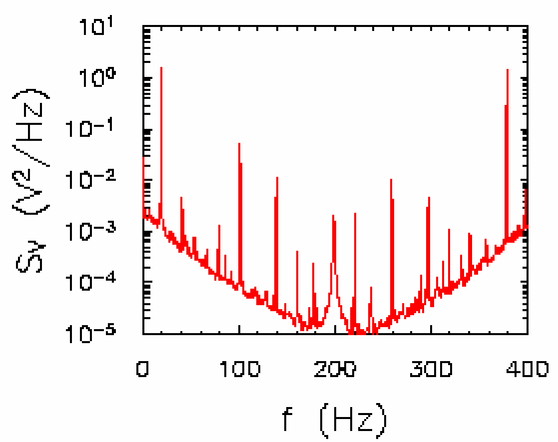

Figure 5. Power spectral density of the output of the multiplier stage. The effect of beating has shifted each spike at frequency f_j in Figure 4 to two new positions, 200 Hz + f_j and 200 Hz - f_j. The desired signal is now located at zero frequency and at 400 Hz. The broad spike now located at 200 Hz is mostly associated with the "1/f

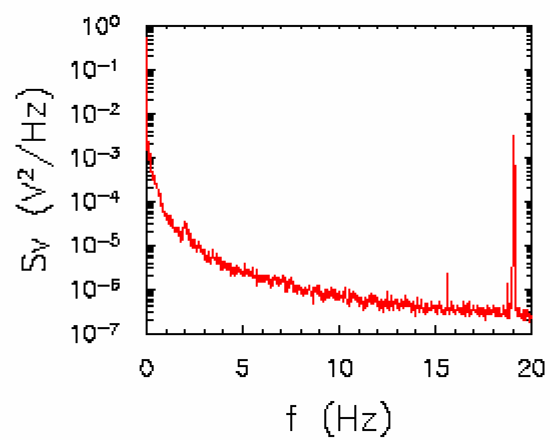

Figure 6. Power spectral density at the output of the DC amplifier. Note the change in frequency scale from the earlier plots. The desired signal at DC is now the only significant signal since the low-pass filter has removed everything above 1 Hz. Though difficult to see on the graph, $S_V(0) = 0.55\ V^2/Hz$.