

Calculation of the dipole mode cavity voltage for the emittance exchange experiment

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Using the thin lens approximation, the condition for a complete transverse-longitudinal emittance exchange is given by [1]

$$1 + k\eta = 0, \quad (1)$$

where η is the dispersion introduced by the dogleg, k is the dipole cavity kick strength. k is related to the cavity voltage by

$$k = \frac{V_0}{aE_b}, \quad (2)$$

where V_0 is the cavity voltage, a is cavity dimension and E_b is the beam energy in the cavity. However, here the definition of a is ambiguous. To clarify, let's write down the general form of TM₁₁₀ mode of a pillbox cavity in the neighborhood of the axis [2]:

$$E_z = E_0 \frac{\omega}{c} \cos(\omega t)x, \quad (3)$$

$$B_y = E_0 \sin(\omega t)/c. \quad (4)$$

where E_0 is the peak electric field. Comparing with Eq. (1) and (2) of Reference [4], we have

$$a = \frac{c}{\omega} = \frac{\lambda}{2\pi}. \quad (5)$$

where λ is the wavelength. Therefore the cavity peak longitudinal voltage is

$$V_0 = \frac{E_b c}{2\pi\eta f}. \quad (6)$$

Let's calculate two examples:

$$\eta = 0.25 \text{ m}, E_b = 5 \text{ MeV}, f = 2.856 \text{ GHz} \Rightarrow V_0 = 0.33 \text{ MV}, \quad (7)$$

$$\eta = 0.25 \text{ m}, E_b = 15 \text{ MeV}, f = 1.300 \text{ GHz} \Rightarrow V_0 = 2.20 \text{ MV}. \quad (8)$$

Another way to look at it [3]

$$\frac{\partial E_b}{\partial x} = \frac{E_b}{\eta} \quad (9)$$

$$\eta = 0.25 \text{ m}, E_b = 5 \text{ MeV}, \Rightarrow \frac{\partial E_b}{\partial x} = 20 \text{ keV/mm}, \quad (10)$$

$$\eta = 0.25 \text{ m}, E_b = 15 \text{ MeV}, \Rightarrow \frac{\partial E_b}{\partial x} = 60 \text{ keV/mm}. \quad (11)$$

so how much transverse kick does this correspond to (or, what is the equivalent transverse voltage) [5]? From reference [4]

$$\Delta x' \approx kz. \quad (12)$$

Imagine an equivalent transverse voltage V_T on the cavity,

$$V_T = \frac{\gamma mc^2}{e} \Delta x' = 0.511 \gamma \frac{z}{\eta} \text{ MV}. \quad (13)$$

e.g., for $z = 1$ mm (the distance of an electron from the center of the bunch), we have $V_T = 60$ kV.

References

- [1] P. Emma, Z. Huang, K.-J. Kim and P. Piot, *Phy. Rev. ST Accel. Beams* **9**, 100702, 2006.
- [2] P. Piot, discussions.
- [3] Tim Koeth.
- [4] M. Cornacchia and P. Emma, *Phy. Rev. ST Accel. Beams* **5**, 084001, 2002. E_z and B_y given by Eq. (1) and (2) are $E_z = E_0 \frac{x}{a} \cos(\omega t)$, $B_y = \frac{E_0}{a\omega} \sin(\omega t)$.
- [5] John Power, private communications.