EX--006
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Effect of thick-lens cavity in dogleg emittance exchanger

Dogleg matrix (from Cornacchia and Emma)

$$M_{DL} = \begin{bmatrix} 1 & 1/2\,L & 0 & \eta \\ 0 & 1 & 0 & 0 \\ 0 & \eta & 1 & 1/2\,\xi \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

L = dogleg length

Thick-lens cavity matrix (from D. Edwards' note)

$$M_{CAV} = \begin{bmatrix} 1 & L_c & 1/2\,kL_c & 0 \\ 0 & 1 & k & 0 \\ 0 & 0 & 1 & 0 \\ k & 1/2\,kL_c & 1/4\,k^2L_c & 1 \end{bmatrix} \tag{2}$$

L_c = cavity length

Total matrix for exchanger

$$M_{TOT} = \begin{bmatrix} 0 & 1/4\,L_c & -1/4\,\frac{L_c+2\,L}{\eta} & -1/8\,\frac{2\,\xi\,L-8\,\eta^2+\xi\,L_c}{\eta} \\ 0 & 0 & -\eta^{-1} & -1/2\,\frac{\xi}{\eta} \\ -1/2\,\frac{\xi}{\eta} & -1/8\,\frac{2\,\xi\,L-8\,\eta^2+\xi\,L_c}{\eta} & 1/8\,\frac{\xi\,L_c}{\eta^2} & 1/16\,\frac{\xi^2\,L_c}{\eta^2} \\ -\eta^{-1} & -1/4\,\frac{L_c+2\,L}{\eta} & 1/4\,\frac{L_c}{\eta^2} & 1/8\,\frac{\xi\,L_c}{\eta^2} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}. \tag{3}$$

Now emittances are mapped accordingly to:

$$\varepsilon_x^2 = |A|^2\varepsilon_{x0}^2 + |B^2|\,\varepsilon_{z0}^2 + \varepsilon_{x0}\varepsilon_{z0}\,\mathrm{tr}\,(U\tilde{U})$$

$$\varepsilon_z^2 = |C|^2\varepsilon_{x0}^2 + |D^2|\,\varepsilon_{z0}^2 + \varepsilon_{x0}\varepsilon_{z0}\,\mathrm{tr}\,(V\tilde{V})$$

note that $\begin{bmatrix} |A| = 0 \text{ and } |D| = 0 \\ |B| = 1 \text{ and } |C| = 1 \end{bmatrix}$ in eqn. (3). U, V are given in Eq. (22) of [Cornacchia + Emma]

$$U = \begin{bmatrix} -1/8\,\frac{L_c\,(-2\,\beta_z+\alpha_z\,\xi)}{\sqrt{\beta_x}\eta\,\sqrt{\beta_z}} & 1/8\,\frac{\xi\,L_c}{\sqrt{\beta_x}\eta\,\sqrt{\beta_z}} \\ -1/8\,\frac{L_c\,\alpha_x\,(-2\,\beta_z+\alpha_z\,\xi)}{\sqrt{\beta_x}\eta\,\sqrt{\beta_z}} & 1/8\,\frac{\xi\,\alpha_x\,L_c}{\sqrt{\beta_x}\eta\,\sqrt{\beta_z}} \end{bmatrix} \quad \text{and} \quad V = -U \quad \text{Eq. 25 [Cornacchia + Emma]} \tag{4}$$

So $tr(U\tilde{U}) = tr(V\tilde{V}) \equiv \lambda^2$

For the matrix in Eq. (3):

$$\lambda^2 = \frac{1}{64}\,\frac{L_c{}^2\,(4\,\beta_z{}^2 - 4\,\beta_z\,\alpha_z\,\xi + \alpha_z{}^2\xi^2 + \xi^2 + 4\,\alpha_x{}^2\beta_z{}^2 - 4\,\alpha_x{}^2\beta_z\,\alpha_z\,\xi + \alpha_x{}^2\alpha_z{}^2\xi^2 + \xi^2\alpha_x{}^2)}{\beta_x\,\eta^2\beta_z} \tag{5}$$

Similarly to [Cornacchia + Emma], look for $\min(\lambda^2)$ w.r.t α_z

$$\frac{\partial\lambda^2}{\partial\alpha_z} = 0 \text{ for } \alpha_z^{min} \longrightarrow \alpha_z^{min} = 2\,\frac{\beta_z}{\xi} \tag{6}$$

and the minimized value for λ^2 is

$$\min(\lambda^2) = \frac{1}{64}\,\frac{\xi^2 L_c{}^2\,(1+\alpha_x{}^2)}{\beta_x\,\eta^2\beta_z} = \frac{1}{64}\,\frac{\xi^2 L_c^2}{\eta^2}\,\frac{\varepsilon_{z0}}{\varepsilon_{x0}}\left(\frac{\sigma_{x'}^2}{\sigma_z^2}\right) \tag{7}$$

$$\begin{cases} \varepsilon_x^2 = \varepsilon_{z0}^2 + \dfrac{\varepsilon_{z0}^2}{64}\,\dfrac{\xi^2 L_c^2}{\eta^2}\left(\dfrac{\sigma_{x'}}{\sigma_z}\right)^2 \Rightarrow \varepsilon_{z0}\sqrt{1+\dfrac{\xi^2 L_c^2}{64\eta^2}\left(\dfrac{\sigma_{x'}}{\sigma_z}\right)^2} = \cancel{\varepsilon_{z0}}\ \varepsilon_x \\ \varepsilon_z^2 = \varepsilon_{x0}^2 + \dfrac{\varepsilon_{z0}^2}{64}\,\dfrac{\xi^2 L_c^2}{\eta^2}\left(\dfrac{\sigma_{x'}}{\sigma_z}\right)^2 \Rightarrow \varepsilon_{x0}\sqrt{1+\dfrac{\xi^2 L_c^2}{64\eta^2}\left(\dfrac{\sigma_{x'}}{\sigma_z}\right)^2\left(\dfrac{\varepsilon_{z0}}{\varepsilon_{x0}}\right)^2} = \cancel{\varepsilon_{z0}}\ \varepsilon_z \end{cases}$$

Numerical estimate of coupling term :

$$(\varepsilon_{x0}, \varepsilon_{y0}, \varepsilon_{z0}) = (3, 10, 3)$$

$$\frac{\varepsilon_{z0}}{\varepsilon_{x0}} \simeq 3$$

$$\frac{1}{2}\xi = 0.06 \text{ m} \qquad \eta = 0.25 \qquad L_c = \frac{2\lambda}{4} + \frac{\lambda}{2} = \lambda = 0.23 \text{ m} \Rightarrow \frac{\xi^2 L_c^2}{64\eta^2} \simeq 2 \cdot 10^{-4}.$$

if transverse waist : $\sigma_{x'}^2 = \frac{\varepsilon_{x0}}{\beta} \Rightarrow \left| \begin{array}{l} \sigma_{x'} \approx 10^{-4} \\ \sigma_z \approx 10^{-3} \end{array} \right.$ assume $\frac{\sigma_{x'}}{\sigma_z} \sim 1$, $\frac{\varepsilon_{z0}}{\varepsilon_{x0}} = 3$

So

$$\varepsilon_x = \varepsilon_{z0} \sqrt{1 + \left\{ \frac{\xi L_c}{8\eta} \left(\frac{\sigma_{x'}}{\sigma_z} \right) \right\}^2} \simeq \varepsilon_{z0} \left[1 + \frac{1}{2} \underbrace{\left(\frac{\xi L_c}{8\eta} \left(\frac{\sigma_{x'}}{\sigma_z} \right) \right)^4}_{(\;)^2 \sim 10^{-4}} \right] \simeq \varepsilon_{z0} (1 + 10^{-8})$$

$$\varepsilon_z = \varepsilon_{x0} \sqrt{1 + \left\{ \frac{\xi L_c}{8\eta} \frac{\sigma_{x'}}{\sigma_z} \frac{\varepsilon_{z0}}{\varepsilon_{x0}} \right\}^2} \simeq \varepsilon_{x0} \left[1 + \frac{1}{2} \underbrace{\left(\frac{\xi L_c}{8\eta} \frac{\sigma_{x'}}{\sigma_z} \frac{\varepsilon_{z0}}{\varepsilon_{x0}} \right)^4}_{(\;)^2 \sim 10^{-3}} \right] \simeq \varepsilon_{x0} (1 + 10^{-6})$$

required tilt: $\alpha_z = \frac{2\beta_z}{\xi} = -\frac{\langle z\delta\rangle}{\varepsilon_z}$

assume a linea chirp in the phase space i.e. $\delta = \delta_{inc} + \frac{\partial\delta}{\partial z} z$,

then $\langle z\delta\rangle = \frac{\partial\delta}{\partial z}\langle z^2\rangle$ so the required phase space slope is: $\frac{\partial\delta}{\partial z} = \frac{-2}{\xi}$

For the forseen dogleg $\xi \simeq 0.1$ so $\frac{\partial\delta}{\partial z} = -20\ m^{-1}$.

The correlation is imparted by operating AWA linac off-crest. Given the gun energy E_0, the energy gain through the linac V the relative energy offset of an electron at position z w.r.t. reference electron is:

$$\delta(z) = \frac{V}{E_0 + V\cos\phi}\left[\cos(kz+\phi) - \cos\phi\right]$$

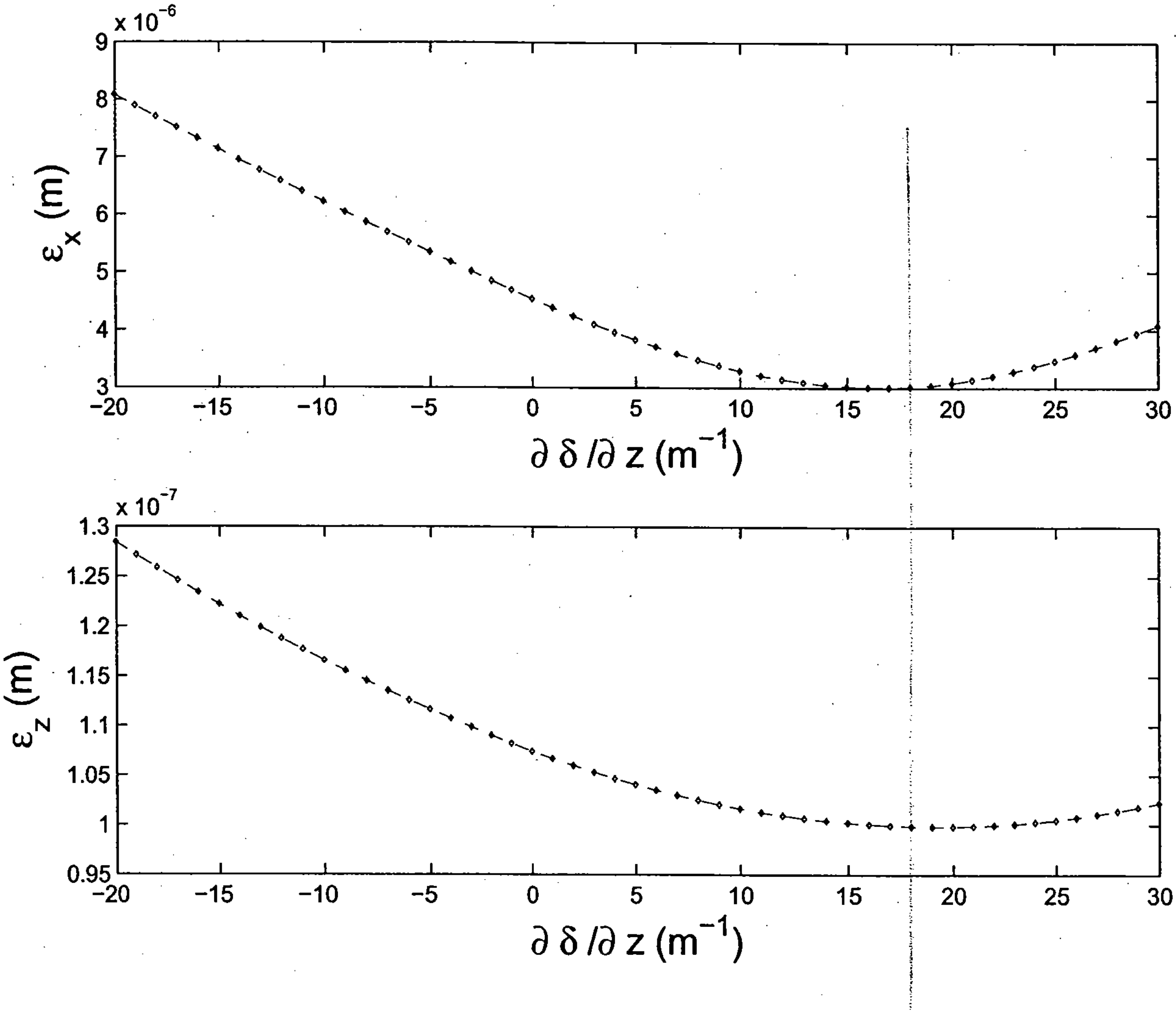
$$\Rightarrow \frac{\partial\delta}{\partial z} = \frac{-Vk\sin\phi}{E_0+V\cos\phi}$$

take $E_0 \approx 8$ MeV $V \approx 8$ MeV $k = \frac{2\pi}{\lambda} \approx 27$ then $\frac{\partial\delta}{\partial z} = -\frac{240}{8}\left(\frac{\sin\phi}{1-\cos\phi}\right) = -30\frac{\sin\phi}{1-\cos\phi}$

$\frac{\partial\delta}{\partial z} \approx -20\ m^{-1}$ for $\phi \approx 5\div6°$

Elegant tracking in the Foreseen.
AWA - dogleg with thick lens cavity.

$\varepsilon_{x0} = 10\ \mu m$, $\varepsilon_{z0} = 3\ \mu m$



optimum chirp close to what estimated page 4. Sign difference due to ELEGANT convention (tail>0).

optimized emittance exchange, tracking in ELEGANT

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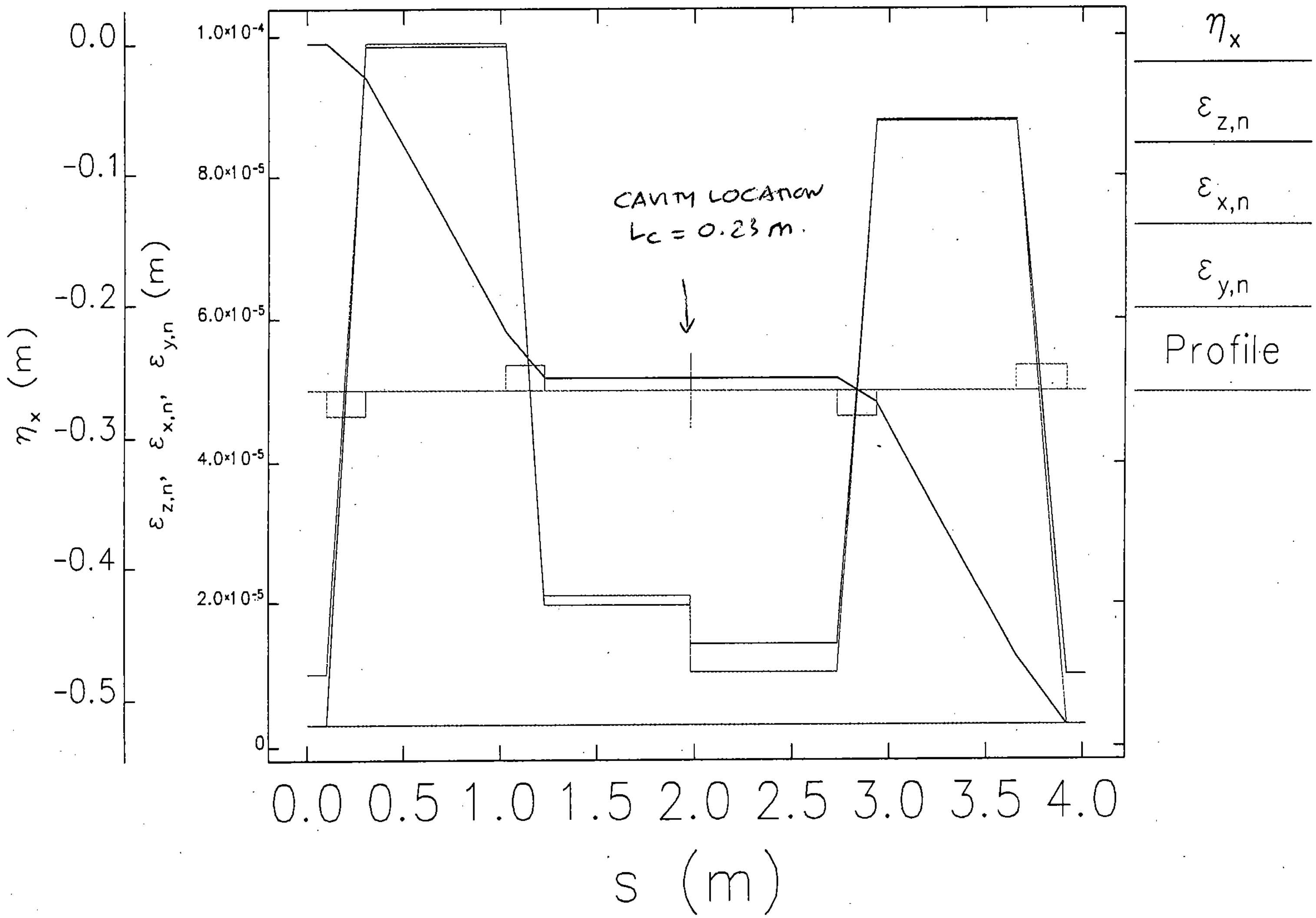

Twiss parameters--input: Xchangeronlythic.ele lattice: Xchangeronlythic.lte