EX-3
March, 10, 2007
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Estimate of the RF Power Required by the Deflecting Mode Cavity

Abstract.

A investigation is being carried out [1] to study the feasibility of performing an emittance exchange (EX) experiment at one of Argonne's two photoinjector-based beamlines: an L-band W_0 = 15 MeV facility or an s-band W_0 = 5 MeV facility. In this note, an estimate of the RF power required by a deflecting mode cavity for each beamline is presented. This RF power estimate is used as a factor in deciding which facility is better suited for the EX experiment and the choice of facility is given at the end of the paper.

I. Introduction.

The EX experiment [2, 3] requires a deflecting mode cavity to make the emittance exchange. Due to the beam pipes attached to the cavity, a real deflecting mode cavity does not support pure TM_{110} or TE_{110} modes but rather supports the hybrid mode HEM_{110} [4]. All the same, a deflecting mode pillbox cavity supporting a standing wave (SW) TM_{110} mode is used as a proxy for the real deflecting mode cavity since it shares many of the same properties and provides a means for a quick estimate of the RF power requirement.

Preliminary simulations of the EX beamline have been carried out by Sun [5, 6]. Using the notation introduced by Cornacchia and Emma [2], Sun estimates the deflecting mode cavity strength parameter T (to be defined in the next section) as $T=4$ *(mrad/mm)*, but it may be as high as $T=8$ *(mrad/mm)*. depending on the exact beamline design chosen. In the remainder of the paper, we first estimate the required field strength, and then estimate the RF power needed to support this field, and conclude by giving the decision of the facility.

II. Estimate of the required field strength, E_0.

A single-cell, standing wave (SW) TM_{110} pillbox cavity is used in the calculations below for both the L-band, operating at f_L=1300 MHz, and the s-band, operating at f_S = 2856 MHz. Since the real deflecting cavity will have multiple cells, then we must choose the operating mode of the accelerating cavities. We will follow the lead of most other deflecting mode cavity designs and chose the π-mode since it has the highest shunt impedance and thus $d=\lambda/2$.

For convenience, we repeat the estimate of Sun [5] with some minor notational differences. Recall Eqn (1) from [1] for the longitudinal field, E_z,

$$E_z(x,y,z,t) = E_0 \frac{x}{a} \cos(\omega t) \qquad \textit{[Ref [2], Eqn (1)]}$$

where ***z*** is the longitudinal axis of the reference trajectory, ***x*** the horizontal axis, ***y*** the vertical, **ω** the frequency of the cavity oscillations, and ***a*** is a constant characteristic of the cavity dimensions. The peak field is $\boldsymbol{E_0 = V_0/d}$, where $\boldsymbol{V_0}$ is the peak longitudinal RF voltage that a particle at horizontal position ***x*** would experience as a function of the RF phase.

Next, we repeat Eqn (4) from [1] for the deflecting mode cavity strength parameter ***T***, in units of [rad/m] or [mrad/mm]. (The symbol in their paper is actually ***k***, but here ***k*** is reserved for the wavenumber),

$$T = \frac{d}{a} \frac{E_0}{(W_0/e)} \qquad \textit{[Ref [2], Eqn (4)]}$$

where ($\boldsymbol{W_0/e}$) is the kinetic energy of the beam in MV and $\boldsymbol{E_0}$ is in MV/m. With these two equations in hand, we can now estimate the required field strength

To determine the value of the cavity parameter ***a***, we write the nonzero field components of the TM_{110} pillbox cavity [4] as,

$$\begin{aligned} E_z(r,\theta,z,t) &= E_0 J_1(kr)\cos(\theta)\cos(\omega t) \\ B_r(r,\theta,z,t) &= -\frac{E_0}{c}\frac{J_1(kr)}{kr}\sin(\theta)\sin(\omega t) \\ B_\theta(r,\theta,z,t) &= -\frac{E_0}{c} J_1'(kr)\cos(\theta)\sin(\omega t) \end{aligned} \tag{1}$$

where $k = \chi_{11}/R$ is the cutoff wavenumber, χ_{11} is the Bessel function root, and ***R*** is the radius of the pillbox. Since the longitudinal mode number is n=0 ($k_z = 0$) then $k = \omega / c$. Next, we expand $\boldsymbol{E_z}$ near the axis as $J_1(x) = x/2$ and use $x = r\cos(\theta)$ so that,

$$E_z(x,y,z,t) = E_0 \frac{kx}{2} \cos(\omega t) \tag{2}$$

which, by matching Eqn 2 to *[Ref [2], Eqn (1)]*, we identify the parameter ***a****=2/k*.

To find the required field strength we solve *[Ref [2], Eqn (4)]* for $\boldsymbol{E_0}$,

$$E_0 = \frac{a}{d} T(W_0/e) \tag{3}$$

or using $d = \frac{\lambda}{2} = \frac{c}{2f}$ and $a = \frac{2}{k} = \frac{2c}{\omega} = \frac{c}{\pi f}$ we have,

$$E_0 = \frac{2}{\pi} T\left(W_0 / e\right) \tag{4}$$

Note that the required field strength, ***E_0***, is independent of the frequency. Physically, this is because the increase in fractional voltage with frequency is offset by the decrease in cavity length.

Using a deflecting mode cavity strength parameter ***T***=4 mrad/mm, ***W_0*** = 15 MeV for the L-band facility and ***W_0*** = 5 MeV for the s-band facility, we have the required field strength as, ***$E_{0,L}$***=38 MV/m and ***$E_{0,s}$***=13 MV/m. A more accurate result is obtained by accounting for the transit time factor, ***TF***. For a pillbox cavity, we guess ***TF***=0.63 (the value for π-mode, TM_{010} cavity) and we finally obtain the estimate of the required field strength,

$$\begin{aligned} E_0 &= 20\, MV/m \quad (s-band) \\ E_0 &= 60\, MV/m \quad (L-band) \end{aligned}$$

II. Estimate of the required field power, ***P****.*

A MathCAD script was written to solve for the required power and the key parts of that script are copied here.

(a) The general expression for the field components of the TM_{qnp} mode [4] are:

$$\mathrm{Ez}(r,\theta,z,t) := \mathrm{Eo}\cdot J_m\left(q, \frac{\chi_{qn}}{a}\cdot r\right)\cdot\cos(q\cdot\theta)\cdot\cos\left(\frac{p\cdot\pi}{d}\cdot z\right)\cdot\cos(\omega\cdot t)$$

$$\mathrm{Er}(r,\theta,z,t) := -\mathrm{Eo}\cdot\frac{p\cdot\pi}{d}\cdot\frac{a}{\chi_{qn}}\cdot\mathrm{DJ}\left(q, \frac{\chi_{qn}}{a}\cdot r\right)\cdot\cos(q\cdot\theta)\cdot\sin\left(\frac{p\cdot\pi}{d}\cdot z\right)\cdot\cos(\omega\cdot t)$$

$$\mathrm{E\theta}(r,\theta,z,t) := -\mathrm{Eo}\cdot\frac{p\cdot\pi}{d}\cdot\frac{q\cdot a}{\chi_{qn}}\,\mathrm{Jdivr}\left(q, \frac{\chi_{qn}}{a}\cdot r\right)\cdot\sin(q\cdot\theta)\cdot\sin\left(\frac{p\cdot\pi}{d}\cdot z\right)\cdot\cos(\omega\cdot t)$$

$$Bz(r,\theta,z,t) := 0$$

$$Br(r,\theta,z,t) := -\frac{Eo}{c}\cdot\frac{\omega}{c}\cdot\frac{a}{\chi_{qn}}\cdot q\cdot Jdivr\left(q,\frac{\chi_{qn}}{a}\cdot r\right)\cdot\sin(q\cdot\theta)\cdot\cos\left(\frac{p\cdot\pi}{d}\cdot z\right)\cdot\sin(\omega\cdot t)$$

$$B\theta(r,\theta,z,t) := -\frac{Eo}{c}\cdot\frac{\omega}{c}\cdot\frac{a}{\chi_{qn}}\cdot DJ\left(q,\frac{\chi_{qn}}{a}\cdot r\right)\cdot\cos(q\cdot\theta)\cdot\cos\left(\frac{p\cdot\pi}{d}\cdot z\right)\cdot\sin(\omega\cdot t)$$

where,

$$DJ(q,x) := \frac{d}{dx}Jn(q,x)$$

$$Jdivr(q,x) := \left|\begin{array}{ll} \frac{1}{x}\cdot Jn(q,x) & \text{if } x > 0.0001 \\ 1 & \text{if } q = 0 \\ \frac{1}{\Gamma(q+1)}\cdot\frac{x^{q-1}}{2^{q}} & \text{otherwise} \end{array}\right.$$

(b) Assuming conductivity for copper of 59.6e6 S/m, then the RF surface resistance is:

$$\sigma_{Cu} := 59.6\cdot 10^{6}\,\frac{S}{m}$$

$$\delta_{Cu} := \sqrt{\frac{2}{\sigma_{Cu}\cdot\mu_0\cdot\omega}}$$

$$R_s := \frac{1}{\sigma_{Cu}\cdot\delta_{Cu}}$$

$$R_s = 9.27958\times 10^{-3}\,\Omega$$

(c) The stored energy, U, is:

$$U := \frac{\varepsilon}{2}\cdot\int_0^{d}\int_0^{2\cdot\pi}\int_0^{a}\left[\left(\left|Ez(r,\theta,z,0)\right|\right)^2 + \left(\left|E\theta(r,\theta,z,0)\right|\right)^2 + \left(\left|Er(r,\theta,z,0)\right|\right)^2\right]\cdot r\,dr\,d\theta\,dz$$

U = 9.26 joule (L-band)
U = 0.097 joule (s-band)

(d) Setting time $= t_{90} = \frac{1}{\omega}\frac{\pi}{2}$, the average power dissipated in the wall is calculated as:

$$P_s := \frac{R_s}{2\cdot\mu^2}\cdot\left[\int_0^{2\cdot\pi}\int_0^d\left[\left(\left|Bz\left(a,\theta,z,t_{90}\right)\right|\right)^2+\left(\left|B\theta\left(a,\theta,z,t_{90}\right)\right|\right)^2\right]\cdot a\,dz\,d\theta\right]$$

$$P_e := \frac{R_s}{2\cdot\mu^2}\cdot\left[\int_0^{2\cdot\pi}\int_0^a\left[\left(\left|B\theta\left(r,\theta,0,t_{90}\right)\right|\right)^2+\left(\left|Br\left(r,\theta,0,t_{90}\right)\right|\right)^2\right]\cdot r\,dr\,d\theta\right]$$

$$P := P_s + 2\cdot P_\epsilon$$

$P = 2.158\ \mathrm{MW}$ (L-band)
$P = 87\ \mathrm{kW}$ (s-band)

(e) The quality factor Q is:

$$Q := \frac{\omega\cdot U}{P}$$

$Q = 35,000$ (L-band)
$Q = 23,600$ (s-band)

Assuming our conductivity is not ideal, then we will only capture 85% of the Q calculated here. Therefore, the power required by a single-cell pillbox-cavity of length **d**=λ/2, operating in a standing wave (SW) TM_{110} mode can finally be given as,

$$P = 2.5\,MW \quad (L-band)$$
$$P = 100\,kW \quad (s-band)$$

III. Choice of facility

While the power required by s-band deflector is about 25x less than the L-band cavity, the absolute power requirement (less than a few megawatts) is modest and well within the RF budget of the L-band facility. Therefore, it was decided [7] that the power requirements alone are not the most important factor. On the other hand, the advantage of performing the E-X experiment with a 15 MeV is significant since the higher energy will stave off space-charge effects which are expected to interfere with the E-X process.

We conclude that the 15 MeV L-band facility is the better choice for the E-X experiment and all future work will concentrate on performing the experiment at the L-band facility.

References

[1] The Argonne Emittance Exchange Webpage. http://aps.anl.gov/Accelerator_Systems_Division/Accelerator_Physics/emitExchange/index.htm

[2] M. Cornacchia and P. Emma, Phy. Rev. ST Accel. Beams 5, 084001, 2002.

[3] P. Emma, Z. Huang, K.-J. Kim and P. Piot, Phy. Rev. ST Accel. Beams 9, 100702, 2006.

[4] T. Wangler, *RF Linear Accelerators*, John Wiley & Sons, Inc., 1998. See pg. 345-346 for deflecting mode; see Chapter 1 for general TM mode.

[5] Y.-e. Sun, Emittance-Exchange Note #1, Dipole Cavity Voltage Calculation

[6] Y.-e. Sun, Emittance-Exchange Note #2, Phase-Space Snapshots

[7] Discussions, K. Harkey, K.-J. Kim, Ph. Piot, J. G. Power, Y.-e. Sun, and W. Gai